CHAPTER 7

Measuring Flexibility

7.1 Introduction

The previous chapter indicated a need to measure flexibility. To make use of the operationalisation of flexibility, i.e. options and strategies to introduce or increase flexibility, we need to assess its costs and benefits. Measuring flexibility facilitates the comparison of options and strategies.

One of the greatest challenge put forth by many researchers is that of unifying definitions of flexibility. There have been as many measures as definitions of flexibility. Mandelbaum (1978) reviewed at least twenty. The previous chapters have shown the difficulty of reconciling its many interpretations and applications. Its multi-dimensional aspects make the quest for a single best measure impractical if not impossible. In the manufacturing sector, where flexibility is a well-recognised and necessary objective, new measures are continually being suggested. There, Sethi and Sethi (1990) and Gupta and Goyal (1989) have provided the most comprehensive classifications and descriptions of different types of measures of manufacturing flexibility up to 1990. Instead of deriving a single measure, Slack (1983) suggests developing a methodology to identify ways and costs of providing any new flexibility. Motivated by these comments, we focus on ***ways of measuring flexibility*** rather than finding a single measure.

In the manufacturing sector, Gerwin (1993) believes that the research on flexibility needs to have a more applied focus to complement the theoretical work, and that the main barrier to advances on both theoretical and applied fronts stems from the ***lack of measures for it*** and ***its economic value***. On the contrary, our review has found a proliferation of measures. Because these measures come from different disciplines, they individually do not reflect the rich cross-disciplinary interpretation of flexibility.

Among others, Bernardo and Mohamed (1992) argue for an explicit consideration of flexibility by definition and classification of measures. Not only is defining and classifying flexibility a major challenge, developing operationalisable measures is also difficult. Gerwin (1993) gives five reasons for this.

1) Little agreement exists on relevant dimensions of the concept. [Our conceptual development has clarified much of the confusion and unified the essential concepts inherent in flexibility.]

2) Multi-dimensionality compounds the effort that goes into creating scales for assessment. [As concluded from our cross disciplinary review and conceptual development, the concept of flexibility is ***multi-faceted***, containing both the context-dependent ***types*** and context-free ***elements*** of flexibility.]

3) Because flexibility can be studied at different system levels, such measures require collections of disparate data sets.

4) Operationalisations that span industries are more useful albeit more difficult to create for research than those based on a single industry. [We have studied the uses and definitions of flexibility from a broad basis, i.e. a cross disciplinary review that spanned industries.]

5) There is a lack of communication between those formalising flexibility concepts and those operationalising them (as in manufacturing flexibility). We address the fifth difficulty by investigating measures of flexibility to tie together the theoretical and practical aspects.

There are other fundamental difficulties in measuring flexibility. Flexibility is evasive because it is a ***potential***, which depends on what happens in the ***future***. However, the future is shrouded in ***uncertainty***, which in the strictest sense of Knight (1921), cannot be measured or predicted with accuracy. Therefore, the value of flexibility is difficult, if not impossible, to ascertain. Pye (1978) even suggested measuring this uncertainty as a substitute for flexibility. In this thesis, we relax the definition of uncertainty to allow a ***probabilistic treatment***.

We propose the following ***criteria for measuring flexibility***. A measure of flexibility should be ***meaningful***, reflecting our conceptual understanding as developed from the cross disciplinary review. As such, there is a need to represent the different aspects of flexibility.

1) It should contain the necessary definitional elements of
   a) ***range***,
   b) ***time***,
   c) ***change***,
   d) ***conditions of uncertainty***, and
   e) ***favourability***.

2) It must not contradict relationships from the ***conceptual framework***.

3) It should be ***simple and operationally possible*** from a modelling perspective, that is, it can be derived or calculated from existing techniques.

4) A measure of flexibility should distinguish between the flexible and the inflexible, and ideally distinguish between different degrees of flexibility.

5) Finally, such a measure should facilitate a ***trade-off*** between the notions of size of choice set and value inherent in flexibility, that is, where flexibility conflicts with favourability.

We classify our extensive review of measures of flexibility into three groups: ***indicators***, ***expected value***, and ***entropy***. Most measures belong to the first group and reflect partial aspects of flexibility. To summarise these partial measures, we translate and organise the necessary definitional elements of Chapter 6 into ***indicators*** of flexibility in section 7.2.

The second group of measures are based on the concept of expected value in decision analysis. Three expected value measures of flexibility are reviewed in section 7.3. 1) The ***relative flexibility benefit*** (Hobbs et al, 1994) puts a positive value on the ability to take advantage of favourable uncertain conditions. 2) The ***normalised flexibility measure*** (Schneeweiss and Kühn, 1990) deals with the states of the uncertain condition matched to the states or choices of the second stage decision, thereby conveying the notion of slack. 3) The expected value of flexibility (Merkhofer 1977, Mandelbaum 1978) relates to ***the expected value of information***. The first expected value measure deals with the number of uncertainties, the second with the states of uncertainties, and the third with the order of decisions and uncertainties.

The third group of measures are based on the scientific concept of ***entropy*** (section 7.4), which has not received enough attention with respect to flexibility. Two types of entropic measures corresponding to the ***decision*** and ***system views*** of entropy are assessed.

Section 7.5 compares expected values with entropy. The final section 7.6 summarises the critique and comparison of these three groups of measures and proposes a way forward.

7.2 Indicators of Flexibility

We give a new terminology for partial measures of flexibility. These measures are translated and organised into ***indicators*** which reflect those necessary definitional ***elements*** of flexibility. The term ***indicator*** is used to ***indicate*** rather than ***measure*** flexibility, as ***measure*** gives the connotation of completeness. An indicator by itself is a partial measure of flexibility.

Centrally embedded in the concept of flexibility is the ***capability*** to change, reflecting a ***potential*** or a ***provision*** for change that is available in the future. Therefore any measure of flexibility should reflect the potential but not necessarily the realisation.

In their analysis of flexibility and uncertainty, Jones and Ostroy (1984) observe that flexibility is a property of an ***initial period position***, referring to the cost or possibility of moving to various ***second period positions***. It has been pictured in Hobbs et al (1994), Hirst (1989), and Schneeweiss and Kühn (1990) as a sequence of decisions in a ***minimum of two stages*** where the first stage is the initial position, providing the flexibility which can be realised in the second stage. It has also been interpreted as a state transition (Kumar 1987 and others), where the initial position can move to another state. Either way, flexibility is associated with the ***initial state*** but measured by the ***number of states*** it can move to or ***the number of choices*** available in the ***second stage***.

The amount of flexibility is ***relative*** not absolute. It is necessary to have a default case for comparison purposes, i.e. the inflexible path with no subsequent options. The choices in the first stage decision lead to different levels of flexibility. An initial position has flexibility if there is ***at least one other state*** it can move to. Similarly, a first stage decision should have in the following second stage decision at least two choices, with the minimum being "change" or "do not change. " The "do not change" is a default or status quo option. A decision sequence that proceeds from the first stage to the second stage "do not change" is similar to staying in the same state.

The ***change*** or ***transition*** is a ***purposeful*** one. In other words, it is neither accidental nor unpredictable. It is induced by a stimulus or a reason. We do not want flexibility for no reason at all, for it is not free. This reflects our understanding that flexibility is a response to uncertainty, complexity, and other motivating forces. We represent this change by mapping the type of uncertainty to type of flexibility, i.e. "***uncertainty-flexibility mapping***." This mapping consists of trigger events, trigger states, decisions, and choices, as explained below.

Change inducers, i.e. relevant uncertainties, are called ***trigger events***, and they determine the type of flexibility that is required. Examples of the flexibility to respond to the trigger event of demand uncertainty include changing production levels, purchasing or selling extra production capability, and holding reserve capacity. The change is the decision maker's response to the trigger event(s) and is represented by a ***decision.*** Trigger events affect flexibility decisions. There may be several events that trigger a second stage decision. Similarly, a second stage decision may consist of several decisions in a sequence.

A trigger event represents an uncertainty that has two or more possible states. For example, demand uncertainty can be high or low. If each state pre-empts or matches a subsequent decision choice, then it is called a ***trigger state*** for that choice. Just as different trigger events pre-empt different types of decisions, different trigger states pre-empt different choices For example, purchasing additional capacity is associated with high demand while selling extra capacity is associated with low demand, with demand being the trigger event. High and low demand are trigger states for the purchasing and selling options, respectively.

Mandelbaum and Buzacott (1990) define flexibility as the ***number of options*** open at the second period. Mandelbaum (1978), Merkhofer (1975), and Rosenhead (1980) support this definition of flexibility, i.e. the size of a choice set associated with alternative courses of action. The ***size of choice set*** can be directly deduced from that meaning of flexibility which relates to "having many choices" and "multi-functionality." It is one of the measures proposed by Marschak and Nelson (1962), who admit that it is insufficient by itself and subject to the partitioning fallacy. In fact, Evans (1982) accuses Marschak and Nelson of confusing the property of flexibility with its measure, implying that the size of choice set is only one aspect of flexibility. Some differentiation of desirability, quality, and diversity among the choices is necessary to avoid triviality, e.g. choices that are feasible but unlikely to be chosen.

According to Upton (1994, p. 77) flexibility is “the ability to change or adapt with little penalty in time, cost, and effort of performance.” Reflecting the difficulties in changing and the barriers to change, these penalties are what Slack (1983) calls “frictional elements.” Reducing the lead time or response time makes it faster to change. Reducing the switching cost makes it cheaper to change. Mandelbaum defines switching cost as the average sum of gains and losses in the transition, which is only incurred if the change occurs. Removing barriers makes it easier to change. We call these indicators ***disablers***.

There is a difference between the cost of providing the flexibility and the cost of changing. The ***enabler*** reflects the cost of providing flexibility, i.e. it guarantees future flexibility and reflects the premium on flexibility. The flexibility associated with any investment or initial position is largely valued by the initial sunk cost commonly known as fixed investment costs or premium of an option. With respect to costs, the enabler and disabler are similar to fixed and variable costs. The enabler is the premium or cost associated with the first decision which guarantees flexibility later on. The disablers indicate the availability of an option by minimal cost, minimal time, and other reduction of barriers. The disabler includes the switching cost when flexibility is realised. The enabler is like Stigler’s (1939) fixed cost, while disablers are variable costs which may or may not occur. These costs are similar to the loss or benefit if the change occurs (Buzacott 1982, Gupta and Goyal 1989) and Mitnick’s (1992) marginal or incremental cost of the additional facility which adds flexibility.

We translate the ***favourability*** inherent in flexibility into positive values which are desirable. We call these elements ***motivators***. These are the benefits or payoffs associated with the choices available.

Mandelbaum (1978) suggests that flexibility can be measured by the ***effectiveness*** and ***likelihood*** of change. But he does not decompose these elements further nor apply them. From our perspective, effectiveness refers to the number of favourable choices, i.e. choices that do actually lead to favourable outcomes. ***Likelihood*** refers to the probability of the occurrence of the trigger state as well as the combined effects of disablers and motivators, indicating the ***probability*** of the subsequent choice being selected. As probabilities of change, these likelihoods also reflect that element of potential. Eppink (1978) associates the likelihood of state transition with the degree of commitment or the decision maker's willingness to abandon his current position. We interpret the likelihood as a function of the disablers (the more difficult the change, the less likely) and the motivators (the better the outcome, the more likely the change.)

The above classification portrays a multi-dimensional picture of flexibility. Flexibility is a ***potential*** or capability to ***change***, associated with an ***initial position***, but measured by the ***number of favourable choices*** that are available later. Favourability is embedded in positive returns or benefits called ***motivators***. The value of future flexibility is captured by ***enablers*** which guarantee that provision of flexibility. The availability of these choices depends on switching costs and other frictional elements called ***disablers***. The type of flexibility depends on the ***type of uncertainty which triggers*** the subsequent choices. The ***likelihood of change*** depends on the probabilities of the trigger states, disablers, and motivators of the choices. Flexibility is relative to other choices in the first stage. Flexibility increases with the number of choices in the second stage, likelihood of favourable choices, and ease of change. Table 7.1 translates essential elements of flexibility into measurable indicators. Although not all are necessary, each indicator is insufficient on its own.

Table 7.1 Elements and Indicators of Flexibility

Elements	Indicators	Representation
Potential, Capability to change	two-stage decision state transition	investment in first stage, characteristics or capabilities in second
Purposeful change, response to stimuli	trigger event trigger state	uncertainty or chance node states of uncertainty
Availability (existence) of transitional states or possibility of change	number of states size of choice set	number of options in second stage
Likelihood of change	probability of trigger state or proportional representation of disablers	probabilities, costs, lead time
Provision, property of initial position, guarantor	enabler	premium, cost of providing the capability
Barriers to change	disablers	switching cost, response time, speed of change, constraints
Favourability	motivators	profit, income, value, benefits

7.3 Expected Value Measures

7.3.1 Relative Flexibility Benefit

Hobbs et al (1994) define flexibility as ***the ability to adapt a system's design or operation to fluctuating conditions***. They propose a measure called the ***relative flexibility benefit*** to capture the benefit or cost-savings in contrasting how well a system performs under a single set of expected future conditions against how well it performs, on average, if all possible conditions and their probabilities are considered. We analyse their example in detail in section 7.3.1.1. Afterwards in section 7.3.1.2, we extend this example to test their claim that the measure can be used to compare investments that differ in the degree of flexibility.

7.3.1.1 Single Investment: Flexible vs Inflexible

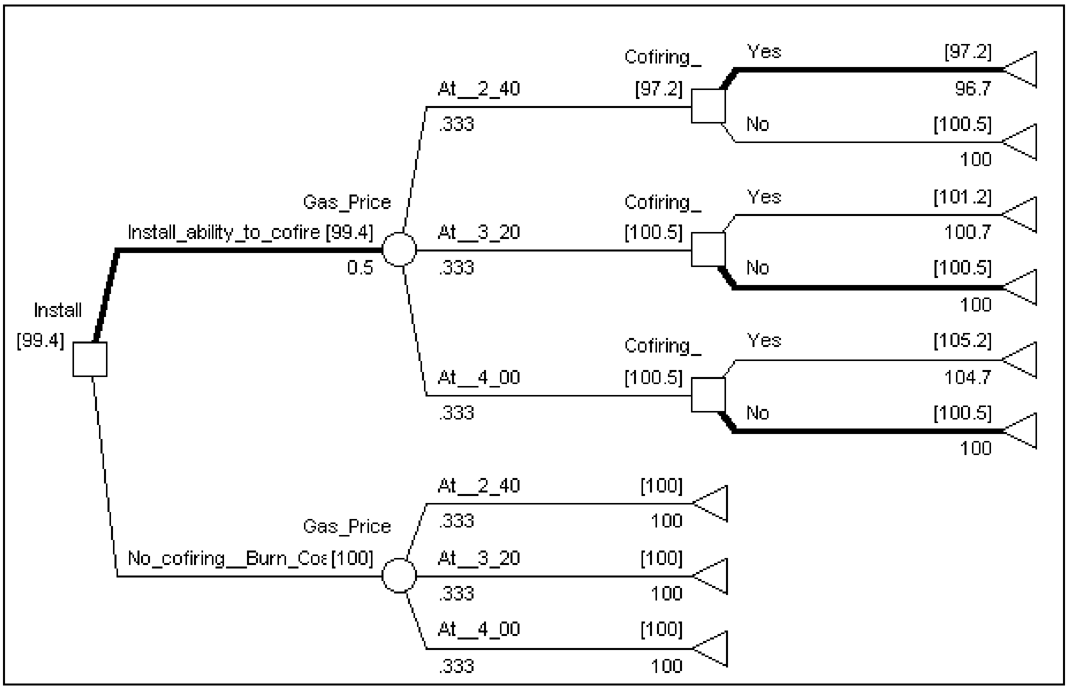
Hobbs et al illustrate their measure through a simple example of a utility's decision to add flexibility to cope with future environmental restrictions. The default option is to ***burn coal***. If co-firing is installed, it can burn either gas or coal. The flexibility of co-firing is merely the additional option of burning gas. Using our terminology of section 7.2, the ***trigger event*** for the flexibility of co-firing is the ***uncertainty of natural gas prices***. The future price of natural gas is equally likely to be one of the following: $2.40, $3.20, and $4.00 per million British Thermal Units. If the capability to co-fire is not installed, the annual generation cost would be $100M, for the default option of burning coal. With co-firing capability at an annual investment cost of $0.5M, the firm can choose to co-fire and pay an annual $96.7M, $100.7M, or $104.7M depending on the price of natural gas. These annual generation costs are added to the annual investment cost to get $97.2M, $101.2M, and $105.2M respectively in table 7.2 and in brackets in figure 7.1. If co-firing is invested but not used, i.e. natural gas prices make it too expensive, the utility firm would incur the annuitised investment cost ($0.5M) plus the default generation cost of $100M, totalling $100.5M. The firm gains flexibility from co-firing because it can burn gas when natural gas price is low and burn coal otherwise, i.e. two choices instead of one. This example treats burning gas and burning coal as mutually exclusive, i.e. there is no intermediate choice of burning both gas and coal.

Table 7.2 Annual Costs

Trigger Event: Natural Gas Price		Second Stage Decision of Co-firing (includes annual $0.5M investment)		No Co-firing (no investment)
State	**Probability**	**Burn Gas**	**Burn Coal**	**Burn Coal**
At $2.4/mmBTU	1/3	$97.2M	$100.5M	$100M
At $3.2/mmBTU	1/3	$101.2M	$100.5M	$100M
At $4.0/mmBTU	1/3	$105.2M	$100.5M	$100M

Figure 7.1 depicts this example in a decision tree. The expected value associated with the portion of the tree to the right of a node, i.e. all branches emanating from the node, is enclosed in brackets above the branches. The direct payoff assignments (annual costs) are given below the associated branches. The expected values associated with end nodes (tip of the right most branches) are totals of the individual payoffs along the path. For example, the total cost of "Burn Gas" given "Gas Prices are $2.4/mmBTU" is [97.2] = 96.7 below the branch + 0.5 install capability to co-fire. All numbers in brackets are calculated by the expected value procedure of averaging out and folding back.

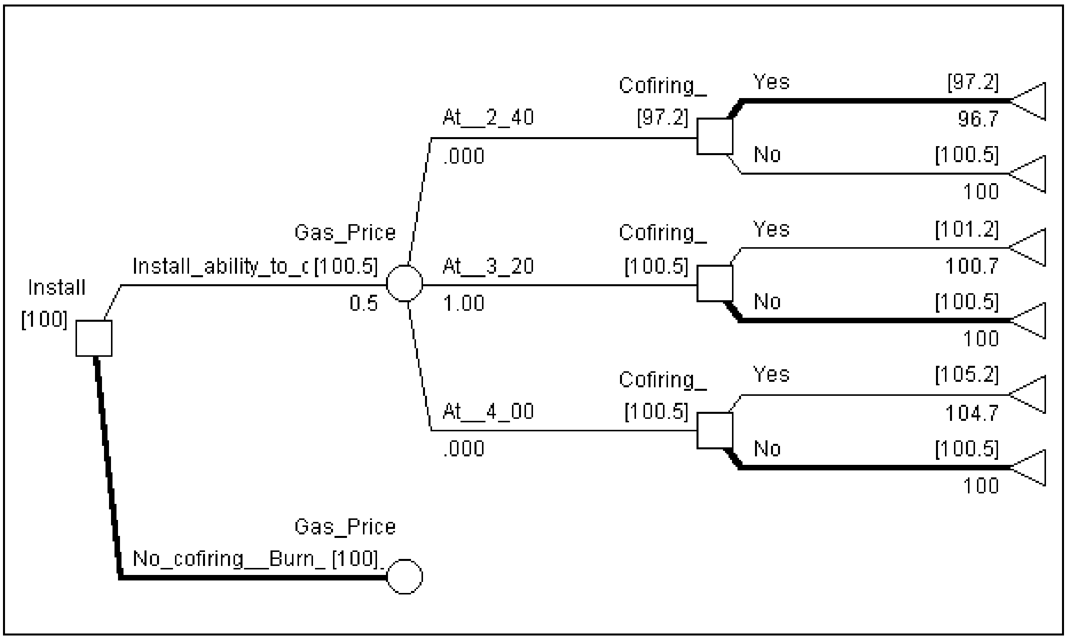
Figure 7.1 Hobbs' Example



To read the decision tree, we start from the left. Invest $0.5M a year to install the ability to co-fire. If the natural gas price is $2.40 (and this occurs with 1/3 probability), the best option is to burn gas and pay a further $96.7M (instead of $100M). If the natural gas price is $3.20 or $4.00, it is cheapest not to burn gas but burn coal instead. If co-firing is installed, one-third of the time it would cost an extra $96.7M to burn gas compared to an extra $100M to burn coal. This gives a savings of $3.3M. [$3.3M = $100M (not installed) - $96.7M (install and run favourably)]. Equivalently, the total annual cost of $100.5M - $97.2M = $3.3M. The benefit of installing co-firing is therefore (1/3) * ($3.3M) = $1.1M cost-savings per year.

Alternatively, this benefit may be calculated by reconstructing the decision tree. The Value of Co-firing Under Expected Conditions minus Expected Value of Co-firing = V (Co-firing | E (Uncertain Gas Price) - E (Co-firing | Uncertain Gas Price). The first term (value of co-firing under expected conditions) is taken from the top portion of the same decision tree with new probability assignments (figure 7.2) which treats the uncertain event as ***certain*** on the ***average state*** and ***improbable under other states***. The second term (expected value (EV) of co-firing given uncertain gas price) is taken from the top portion of the decision tree in figure 7.1. Under expected conditions, the price of gas is 1/3 * ($2.4) + 1/3 * ($3.2) + 1/3 * ($4.0) = $3.2/mmBTU. To get the Value of Co-firing given Expected Conditions, we reassign the $3.2/mmBTU to 100% probability of occurrence. All other prices are set to 0% probability for the purposes of expected value calculation. [Note: This re-assignment of probabilities is possible because the original probabilities were equal. If the original probabilities were not equal, we may require a new state to represent the average.] In this example, the values V and expected values E refer to costs and expected costs, respectively, in which case, the best payoff comes from not burning gas. The benefit or cost-savings of co-firing is thus $100.5M - $99.4M = $1.1M.

Figure 7.2 Expected Conditions



To avoid confusion, we distinguish between the ***value of installing the capability to co-fire*** and the ***value of co-firing***. The value of installing the capability to co-fire is the difference between the expected value of the top branch (install) in figure 7.1 and the expected value of the bottom branch (no co-firing) less the investment cost. In cost terms, -$99.4M - (-$100M) - $0.5M = cost savings of $0.1M. The value of co-firing depends on the price of natural gas. In other words, the value is highest when gas is cheapest, and lowest when gas is most expensive.

Hobbs' ***relative benefit of flexibility*** is the difference between the expected value from considering the uncertainty of natural gas prices and that from treating natural gas prices as one average value. It is the difference between considering several futures (three states of the world) and one representative future (one average state of the world.) The bottom portion of the decision trees (No_co-firing) serves as the default case for comparison purposes, as flexibility is a relative value. The value of flexibility is relative to the zero value of the default inflexible case. It shows that the default case is not affected by the future price of natural gas.

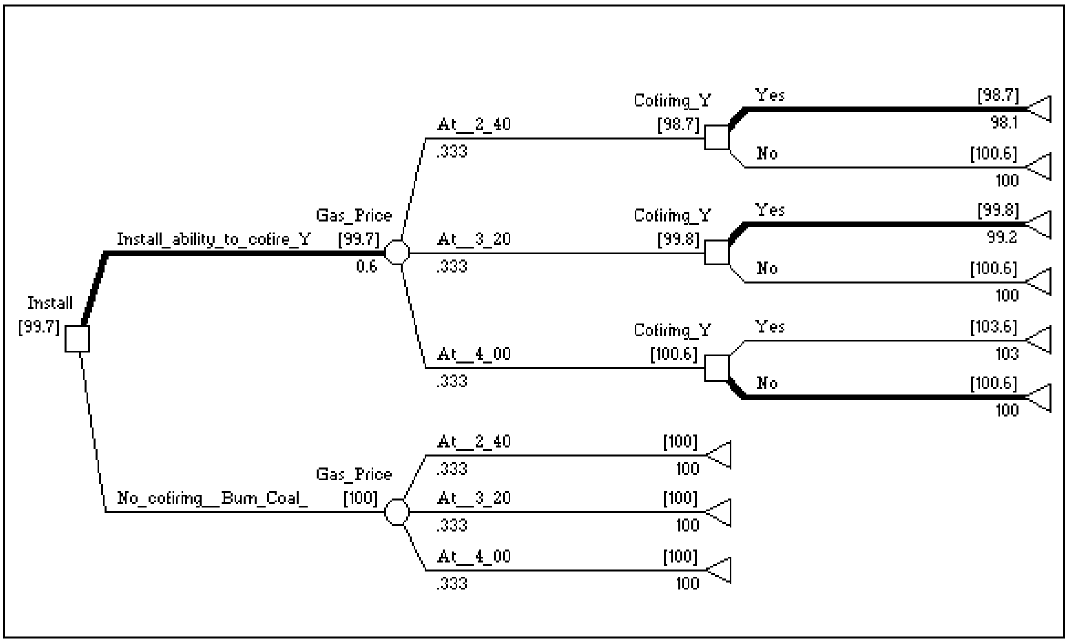
The above analysis agrees with our understanding that flexibility is a potential and a property of the initial position. When we speak of the flexibility of co-firing, we mean installing the capability to co-fire. We measure this potential by a probabilistic weighting of the optimal realisation, i.e. burn gas when natural gas price conditions are favourable. This flexibility is only favourable when the natural gas price falls to $2.40/mmBTU, which occurs a third of the time.

7.3.1.2 Comparing Investments: Flexibility vs Favourability

Hobbs et al claim that this relative flexibility benefit can be used to compare two different investments offering different degrees of flexibility. The relative flexibility of X compared to Y is simply the difference between them, i.e. F(X) - F(Y), where F(X) = V(X | E(uncertain condition)) - E(V(X| uncertain condition)).

We test their assertion by considering two co-firing investments X (described in the previous section) and Y (described below). An alternative co-firing investment Y gives cost-savings (compared to the No_co-firing case) for two out of three natural gas price levels. Y is more flexible than X as it offers one more favourable option than X does. As a result it costs more to invest in Y's capability, $0.6M per year (instead of $0.5M for X). Figure 7.3 shows the relevant decision tree for Y.

Figure 7.3 **Investment Y**



Although Y gives favourable payoffs ($98.7M and $99.8M) more often than X does, its expected cost ($99.7M) is greater than X's ($99.4M). The value of co-firing Y under expected conditions is $99.8M. Its flexibility benefit ($0.1M = $99.8M - $99.7M) is much lower than F(X) which was $1.1M***. Y is more flexible but less favourable than X.*** However, on grounds of flexibility, the firm should favour Y because it is ***more likely*** than X to give cost-savings. On average, Y would be 2/3 favourable compared to 1/3 for X. The flexibility benefits F(X) and F(Y) indicate incorrectly that X gives more benefits of flexibility than Y.

This measure of relative flexibility benefit over-emphasizes ***favourability*** and neglects other aspects of flexibility, e.g. ***likelihood*** and ***number of choices***. Favourability refers to value optimisation, e.g. maximising profit or minimising costs, and uncertainty is not required for its assessment. In this example, favourability refers to minimising annual costs. Flexibility refers to having choices to increase the likelihood of reaching a favourable outcome in the future. X has one out of three chances to give a favourable payoff of $97.2M, while Y has two out of three chances ($98.7M and $99.8M). Although Y's payoffs are not as favourable as X's, Y "seizes more opportunity" and offers a larger (favourable) choice set than X. Therefore Y is more flexible by this definition. However, in one of the three cases, X gives greater cost-savings than Y.

Why have the authors confused flexibility and favourability? Their argument is sound: if flexibility is desirable, then it should give benefits. The more flexible it is, the more benefits it should give. Expected value is a good candidate for the measure of flexibility because it incorporates uncertainty as well as value. Their relative flexibility benefit fails when comparing two investments that conflict on flexibility and favourability, i.e. X is more favourable (more cost-savings) but less flexible (fewer favourable choices) than Y. The relative flexibility benefit measure fails to distinguish between the two. In more complicated examples where a multitude of uncertainties and decisions prevail, the relative flexibility benefit could lead to a poor recommendation. Hobbs et al have assumed that the more flexible it is, the more favourable it is, i.e. no conflict between the two. In doing so, they have disregarded Mandelbaum and Buzacott's (1990) work on flexibility and favourability as two separate decision criteria.

Hobbs et al do not emphasize enough the importance of the uncertainty of natural gas prices. This is the trigger event, without which flexibility cannot be valued. They also fail to mention the implications of the number of trigger states and their probability distribution, i.e. the gas prices. If \$2.40/mmBTU is most likely, then installing X gives the best favourability and flexibility. If \$3.20/mmBTU is most likely, then Y gives the best outcome. If \$4.00/mmBTU is most likely, then co-firing capability is not valuable, as natural gas becomes too expensive for co-firing. The trigger event (natural gas price uncertainty) is an indicator of flexibility. The trigger states are indicators of specific options.

Defining flexibility as the number of choices and favourability as optimising the final payoff, we can reduce this problem to the following. X gives one out of three favourable options in the second stage. Y gives two out of three. Thus Y is the most flexible. On favourability, X has a better expected value, so it is most favourable. If we choose Y, we optimise on flexibility and satisfice on favourability. If we choose X, we optimise on favourability and satisfice on flexibility. A measure of flexibility must give indication of likelihood, which has been lost in expected values.

The likelihood of reaching a favourable outcome is embedded in expected values but averaged out when comparing X and Y. The number of favourable choices is not that which is offered by the co-firing option, i.e. burn gas or not, but in context of natural gas price uncertainty. So X actually offers fewer favourable options than Y if there are three equi-probable prices. A cumulative probability notation may be more appropriate than counting the number of choices.

A decision tree analysis, without examining the structure of the tree itself but only looking at expected values, will gloss over this. This of course depends on the firm's view of natural gas price uncertainty, being only three states and of equal probability in this example.

7.3.2 Normalised Flexibility Measure

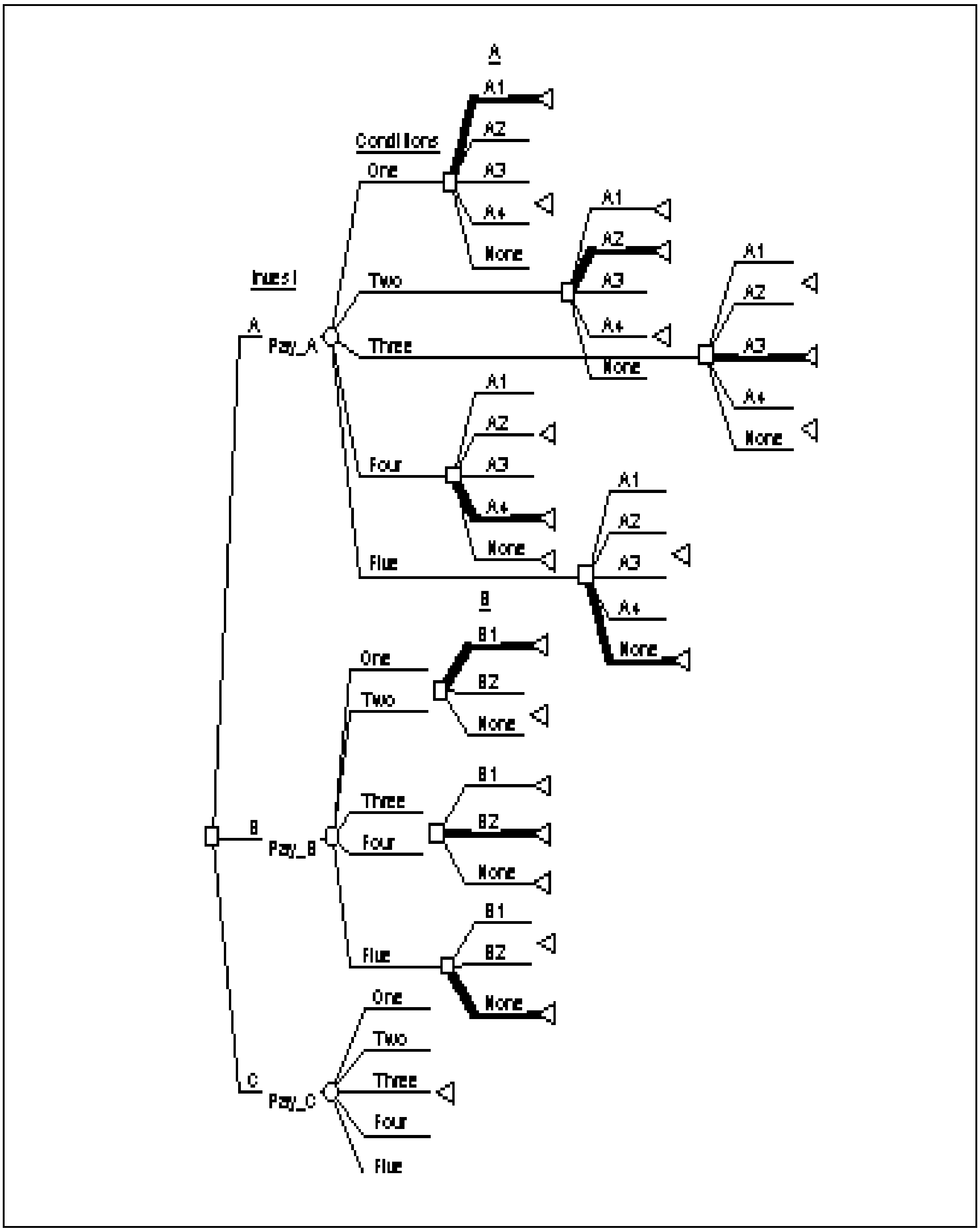
The relative flexibility benefit is unable to distinguish between investments with different degrees of flexibility. The ***normalised flexibility measure*** of Schneeweiss and Kühn (1990) is intended for the comparison of more than two options. It is a ratio of differences of expected values. The denominator is the difference between the ideally flexible V^+ and the most inflexible V^- options considered in the analysis. The numerator is the difference between the chosen investment V^* and most inflexible V^-. So, $N(V^*) = (V^* - V^-) / (V^+ - V^-)$. A normalised measure provides an index between 0 and 1 to rank different options. The problem is finding appropriate values for V that capture both flexibility and favourability for use as anchors in the denominator.

The most flexible option is the first stage option which gives the most number of choices in the second stage. Each choice (available in the second stage) meets each uncertain condition favourably, i.e. gives the best payoff provided that the trigger or favourable state occurs. The choice that gives the most flexibility is able to meet (or take advantage of) each uncertain condition exactly. The less flexible options do not meet the uncertain condition exactly.

One choice corresponds to the most inflexible case (C in figure 7.4), that is, where uncertain conditions cannot be dealt with at all in the default case, i.e. there are no future options available. Another choice represents the ideally flexible case (A) which may or may not appear in the problem, as long as its value is known. The ideally flexible case represents perfect mapping between states of uncertainty and choices of the decision. The remaining choice (B) has a degree of flexibility between the two extremes. Any path that gives some flexibility, i.e. more than one option in the second stage, has a value between the two extremes. Thus the normalised flexibility measure of the chosen branch (B) from the first decision is

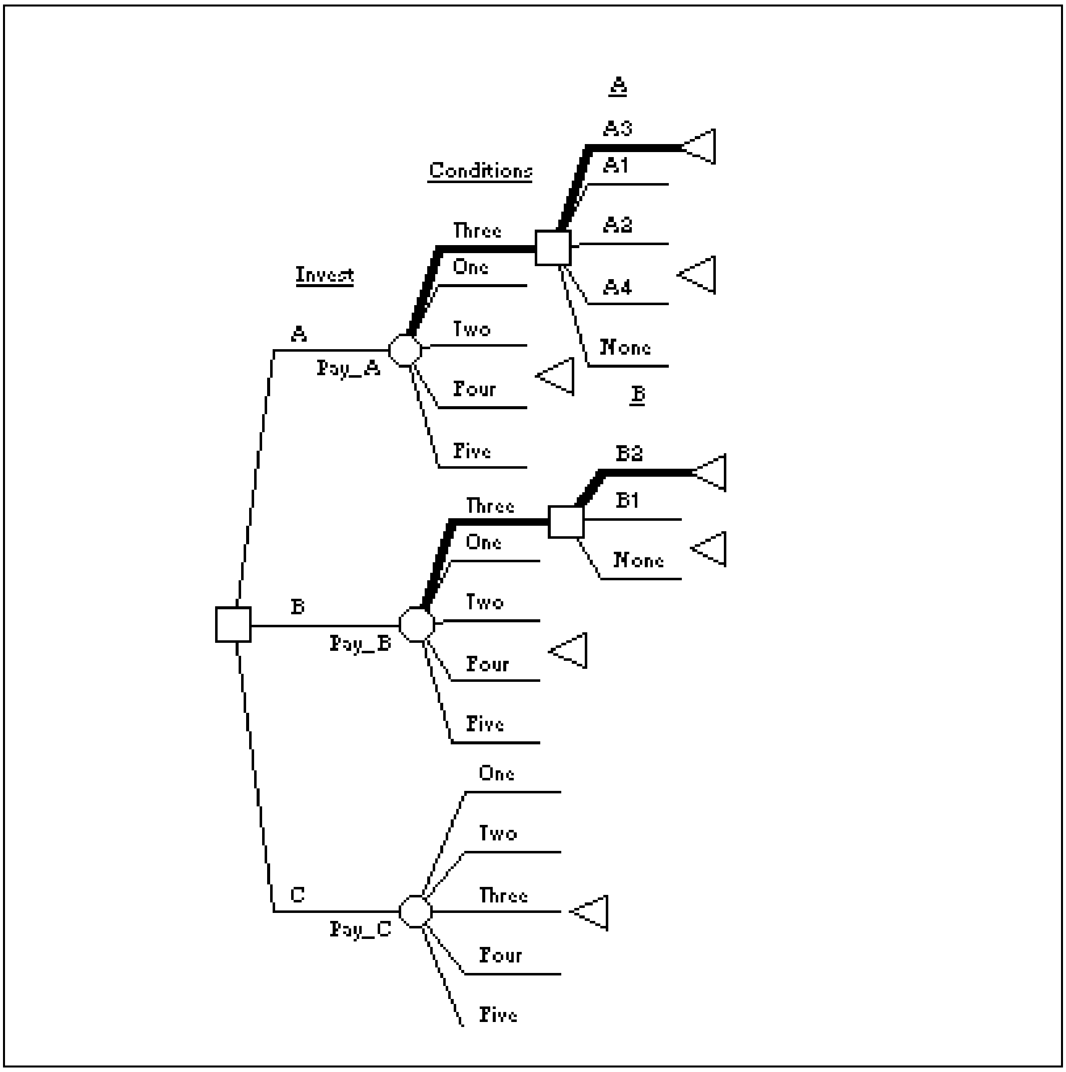
N(B) = [E (chosen) - E (inflexible)] / [E(ideally most flexible) - E(inflexible)]. The most flexible is N(A) = 1 and most inflexible is N(C) = 0. This type of flexibility has to do with matching of trigger states to choices in the second stage decision. It can be generalised to figure 7.4 below.

Figure 7.4 **General Structure of Normalisation**



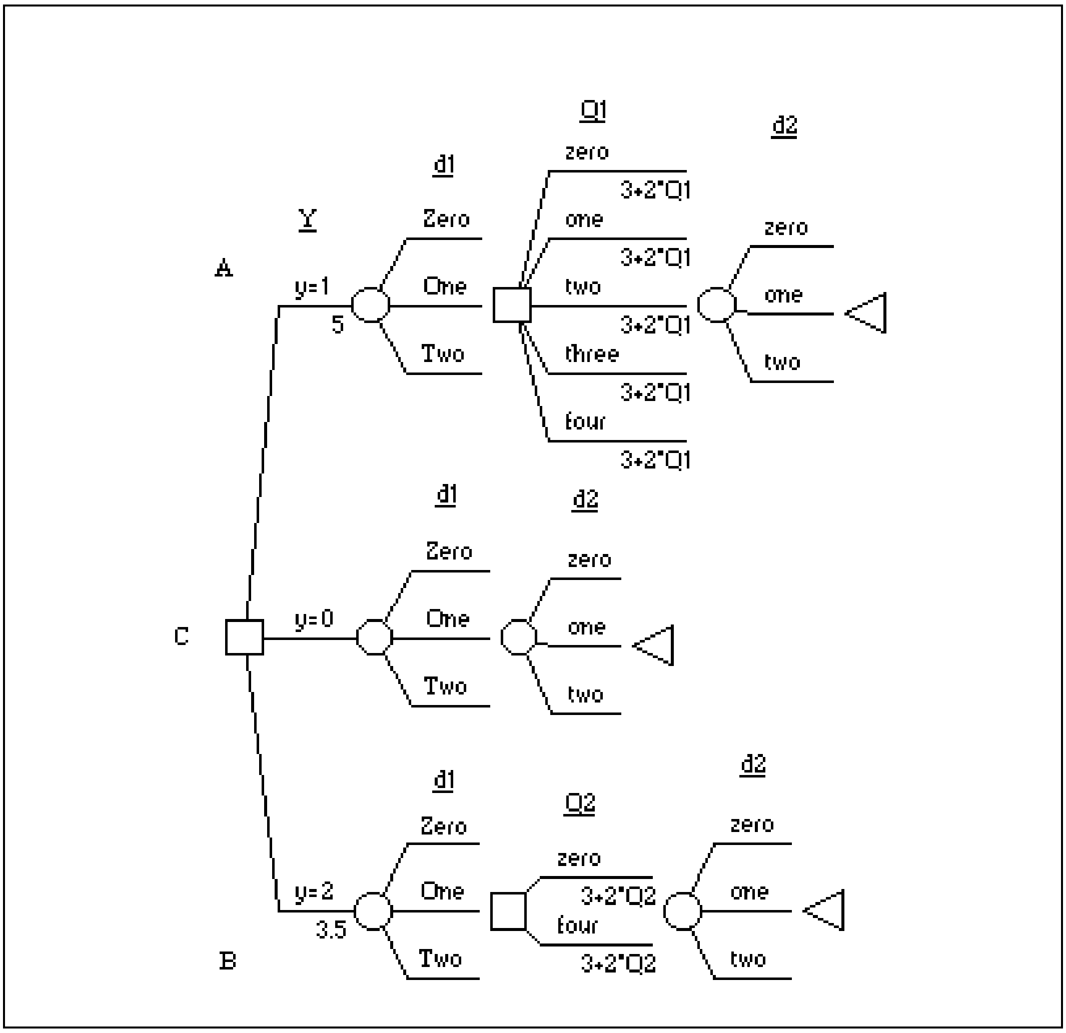
Using Hobbs' flexibility benefit measure, we would get figure 7.5 for the values given expected conditions. That is the E(Conditions) = branch Three with 100% probability, and A3 is the option that gives the most favourable outcome or B2 if B is invested.

Figure 7.5 Expected Conditions



Schneeweiss and Kühn's example concerns machines A and B with different production level settings to meet different demand levels, with C as the default "do nothing case," in which case Pay C = 0. Pay A and Pay B are enablers. The machine with the most number of levels minimises slack production in meeting demand $\boldsymbol{d_1}$ and $\boldsymbol{d_2}$ in figure 7.6. Theirs is one of constrained optimisation, adjusting production levels to meet demand, otherwise incurring a shortage cost. So flexibility is embedded in the notion of minimising slack. The more options (production levels) available, the better is the match.

Figure 7.6 Schneeweiss and Kühn



In their example, there is no second stage decision following the inflexible option of C, implying that the payoffs cannot be adjusted like A and B. The expected values of A, B, and C options include the amount of "slack" arising from the imperfect mismatch of supply and demand. The normalised flexibility measure permits the comparison of a variety of machines and situations commonly found in the manufacturing sector. For example, Röller and Tombak (1990) use machines capable of producing different products to cater for different demand requirements. In electricity capacity planning, this generic example extends to capacity levels, loading order (black-start capability), and unit size as they correspond to demand levels and load distribution. This measure is highly dependent on the partitioning and specification of the states of the trigger event and mapping to the second stage options. Implicitly, the more options available, the better is the match. As decisions are discrete, in the limit, partitioning approximates to the continuous case of meeting different states of uncertain conditions, thereby reducing slack (and giving a higher normalised flexibility measure.) As the trigger event must be the same for all options considered, this measure cannot be used to assess different types of flexibility.

7.3.3 Expected Value of Information

According to Merkhofer (1975), flexibility is valuable (and thus desirable) when new information can be obtained, i.e. one expects to "learn" about the future. The value of this flexibility is directly related to the value of this information.

In decision analysis, the ***value of information*** is the *most a decision maker would pay to resolve an uncertainty, effectively the price of advance information*. This is normally calculated by reconstructing the decision tree such that all uncertainties occur before the decision nodes, so that appropriate action may be taken to optimise payoffs. The difference between the "flipped" tree and the original tree is the value of information. Mandelbaum (1978) says the ***Expected Value of Flexibility*** is the most that a decision maker is expected to gain by delaying decisions. [Note: Delaying decisions is only one strategy for increasing flexibility.] It is the maximum a decision maker can gain by taking the flexible initiative. He associates a given amount of information with a given amount of flexibility.

In a related application, Merkhofer (1977) suggests a reordering of decision and chance nodes to manipulate the amount of decision flexibility. He defines ***decision flexibility*** as the *size of choice set associated with a decision*. [Note: The size of choice set is only one element of flexibility.] The value of information, that is, resolving the uncertainty before making the decision, depends on the amount of decision flexibility. This can be interpreted as follows: the usefulness of putting the chance node before the decision node depends on the degree to which the subsequent payoff can be favourable (instead of unfavourable). Re-ordering of these nodes is achieved by delaying the decision until uncertainty is resolved or obtaining advance information to resolve the uncertainty before the decision is made. Either way, there is a cost to uncertainty resolution, and this is equivalent to the value of information.

Merkhofer's Expected Value of Perfect Information Given Undiminished Flexibility (EVPIGUF) is the upper limit to the value of information given some level of decision flexibility, just as the Expected Value of Perfect Information (EVPI) computed from "flipping" a decision tree gives the upper limit to information gathering. [Note: The upper limit to information gathering is not the upper bound to maximum flexibility.] His measure is most relevant when a decision maker has several decision variables to manipulate, i.e. several decisions to make, the order and the timing of which can be adjusted to allow the resolution of uncertainty beforehand.

The trigger event acts as perfect information for the subsequent second stage decision. Hobbs' relative flexibility benefit is merely the difference between the expected value of perfect information and value of expected information. Perfect information is the exact, actual, and zero error value or prediction. Figure 7.7 illustrates the calculation of EVPI, which has similarities with Hobbs' relative flexibility benefit in figure 7.8.

Figure 7.7 EVPI

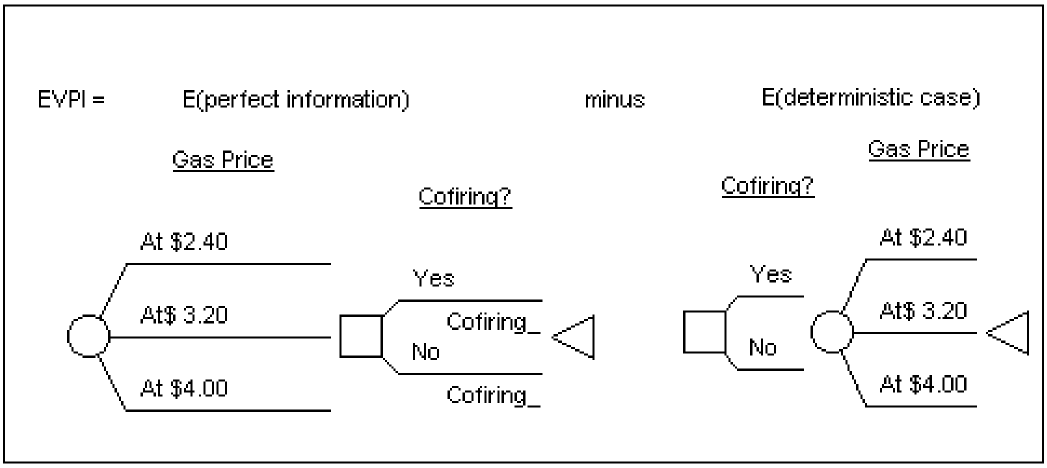
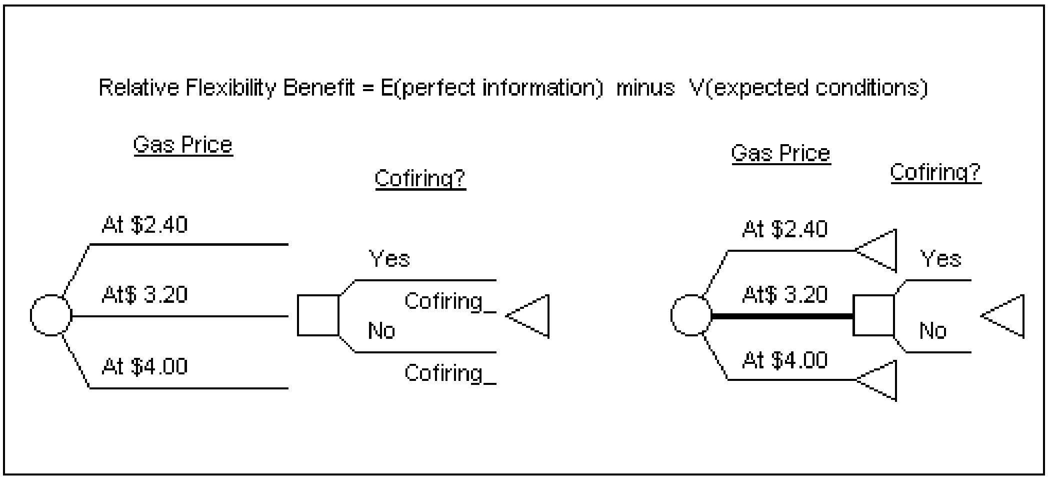


Figure 7.8 Relative Flexibility Benefit



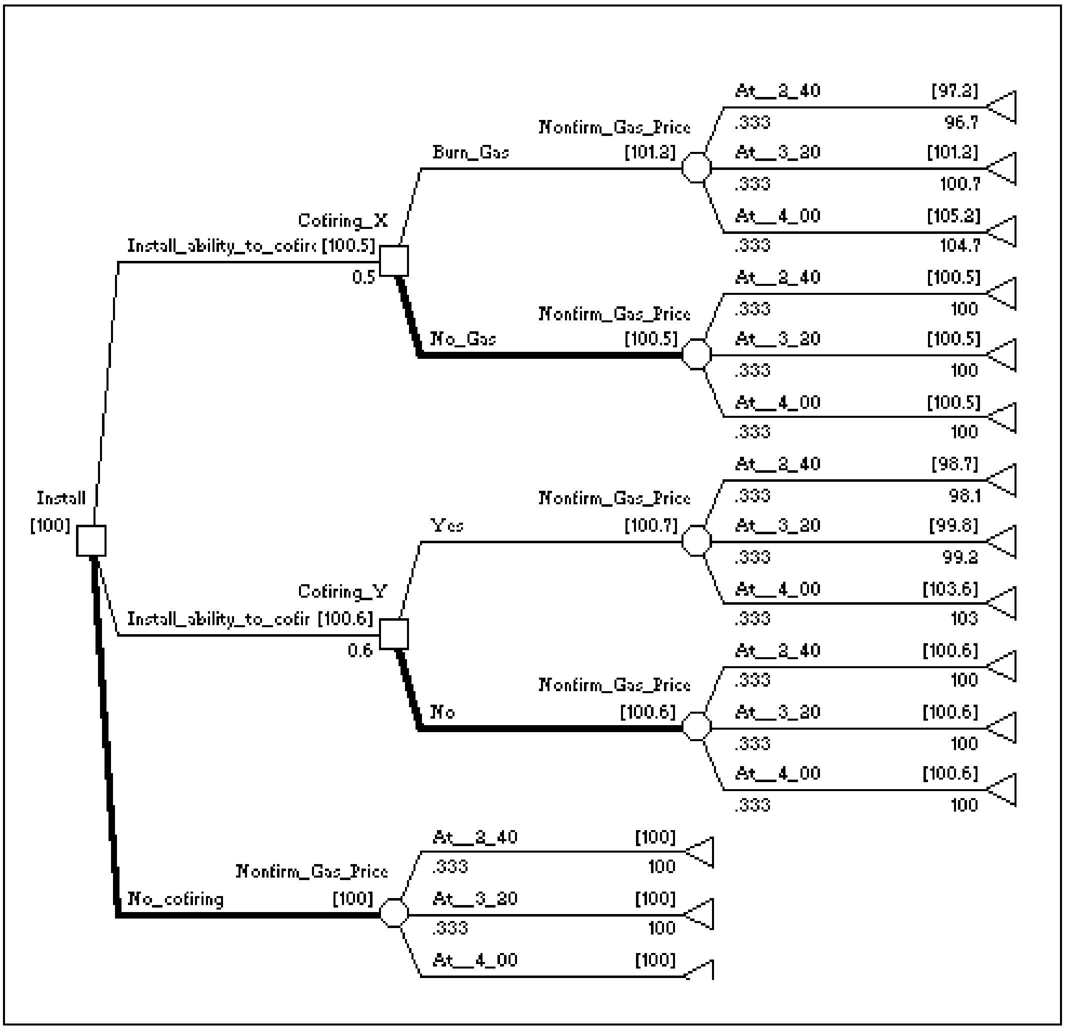
Mandelbaum's and Merkhofer's suggestions of using the expected value of information is confined to that aspect of flexibility that corresponds to the number of choices and that operationalisation of flexibility that is achieved by postponing decisions. Further tests are required to determine its generalisability. In the next section, we explore the possibility of combining features of expected value measures to improve upon the overall flexibility measure.

7.3.4 Towards an Improved EV Measure

We propose and test two ways of improving upon the expected value measures found in the literature. First, we propose to recalculate the normalised flexibility measure using ***EVPI*** and ***Deterministic EV*** as anchors in the denominator. Second, we propose to ***weight*** this new normalised measure by ***the cumulative likelihood*** of the ***favourable trigger states***. These measures are tested and compared against existing expected value measures.

Putting all uncertain events before all decisions gives the highest expected value associated with the maximum flexibility possible with postponement of decisions (EVPI). Putting all uncertain events after all decision sequences gives the expected value associated with zero decision flexibility (Deterministic EV). The Deterministic EV configuration is given in figure 7.9. Since the EVPI and Deterministic EV bound the reasonable spectrum of expected values (from the most inflexible to most flexible), they are candidates for anchoring Schneeweiss and Kühn's normalised flexibility measure. Any intermediate configuration in which decisions and uncertainties are reordered should give an overall expected value that falls between the EVPI and Deterministic EV. Expected values that fall outside the range defined by the Deterministic EV and EVPI are not worth considering as these options are too unfavourable compared to the flexibility they offer. In the flexibility decision tree configuration, both X and Y provide expected values between EVPI and the Deterministic EV, thus indicating that X and Y provide some degree of flexibility.

Figure 7.9 Deterministic EV



The flexibility of X and Y depends on the number of favourable choices which in turn depends on the likelihood of the trigger states of the trigger event (natural gas price uncertainty.) This suggests that expected values should be weighted by the probabilities of the favourable outcomes. We compare the effects of assigning different probabilities of natural gas prices on subsequent expected value measures of the extended example in section 7.3.1.2 in table 7.3.

Table 7.3 Comparison of Expected Value Measures

Scenario	A	B	C	D
Natural Gas Price: P($2.4); P($3.2); P($4.0)	1/3; 1/3; 1/3	1/5; 2/5; 2/5	3/5; 1/5; 1/5	1/5; 1/5; 3/5
Average Gas Price	$3.20	$3.36	$2.88	$3.52
V(X|E(gas price))*	100.5	100.5	97.2, 100.5	100.5
E (X) in Flexibility	99.4	99.84	98.52	99.84
F(X) = V(|E) - E (X)	1.1	0.66	-1.32, 1.98	0.66
V(Y|E(gas price))*	99.8	99.8, 100.6	98.7, 99.8	99.8, 100.6
E (Y) in Flexibility	99.7	99.9	99.3	100.06
F(Y) = V(Y|E) - E (Y)	0.1	-0.1, 0.7	-0.6, 0.5	-0.26, 0.54
EVPI	99	99.36	98.28	99.4
Deterministic EV	100	100	99.6	100
Normalised X	0.6	0.25	0.818	0.267
Normalised Y	0.3	0.1563	0.227	-0.100
Weighted Normalised X	0.2	0.0500	0.4909	0.053
Weighted Normalised Y	0.2	0.0937	0.1818	-0.040

**Lacking more information, we take boundary values rather than interpolating.*

Average gas price is computed by summing the gas prices weighted by probabilities. For example, $3.20 = 1/3 ($2.4) + 1/3 ($3.2) + 1/3 ($4.0). V(X|E(gas price)) refers to the value of the best choice for second stage X given the average gas price. The relative flexibility benefit gives negative values in some instances. This is misleading as a different probability assignment should not change the scale of benefits so dramatically.

The EVPI is the expected value of the decision tree with the natural gas price uncertainty resolved before the decisions are made. The Deterministic EV refers to the expected value of the decision tree under the deterministic configuration in figure 7.13, i.e. all decision nodes followed by all chance nodes, not Hobbs' value given expected conditions. The difference between the EVPI and Deterministic EV serves as a normalising denominator. EVPI is the upper bound to flexibility. Deterministic EV reflects the EV of the most inflexible case. "EV (X) in Flexibility" is the expected value of investing in X in the flexibility configuration, i.e. investment decision followed by uncertainty followed by the second stage decision of burning gas or burning coal. The normalised measure is the difference between the expected value of the particular investment and the deterministic EV divided by the normalising denominator.

The ***weighted measure*** is simply the product of the probability of favourable conditions and the normalised measure. The weighting indicates ***how likely*** or, more appropriately, ***how often*** the normalised measure is expected. This says that if the three different natural gas prices are equally likely to occur (probability of 1/3), then X and Y are equal (0.2) or incomparable. In this scenario, we cannot distinguish between the two investments on the basis of this measure. If the probability assignments are 1/5, 2/5, and 2/5, this means that X will be favourable 1/5 of the time and Y will be favourable 3/5 of the time. The normalised measure does not indicate this, but the weighted measure does. In this case, the weighted measure clearly indicates that Y is preferred, because it capitalises on favourable outcomes. On the other hand, if the probabilities are distributed 3/5, 1/5, and 1/5, X would be the preferred investment as cost-savings outweigh the chances that Y could make. If the probability assignment shifts in the direction of the most expensive natural gas price, where neither X nor Y could give favourable payoffs, the normalised measure for Y becomes negative. Thus Y should not be considered as the choice between $103.6M burn gas and $100.5M burn coal (given co-firing is installed) is altogether unfavourable.

In this particular example, the weighted normalised measure outperforms the relative flexibility benefit and the normalised flexibility measure. It correctly ranks investments differing on degree of flexibility and favourability that are included in the same decision tree. The weighted measure is more meaningful than the normalised flexibility measure. It is not misleading like the relative flexibility benefit measure. A zero or negative measure means that any flexibility provided by the associated investment is outweighed by its negative favourability.

Our proposed weighted normalised measure has some weaknesses.

1) It may not be possible to re-order the nodes so that all chance nodes occur before all decisions to get the EVPI. These are due to the conditionality expressions defined in the corresponding influence diagram. Thus we do not get the true EVPI as the upper bound but a lower EV of less than perfect information.

2) Similarly, it may not be possible to put all chance nodes after all decision nodes to get a Deterministic EV. The inflexible or status quo initial option may not be most favourable. The deterministic EV does not necessarily refer to the worst expected value, only that the inflexible case is the best because uncertainty is not considered and no premium is included in its cost.

3) Weighting by the proportion of favourable states may also invite criticism. The weighting does not reflect the degree of favourability, but only distinguishes between the favourable and the not, i.e. the triggered and untriggered states.

7.4 Entropic Measures

The close relationship between uncertainty and flexibility suggests the use of entropy to measure flexibility. Entropy originates from the Greek word meaning ***transformation***. Entropy is a logarithmic measure of ***uncertainty*** in information theory, ***randomness*** in nature, ***disorder*** in thermal dynamics, ***information*** in cybernetics, ***concentration*** in economics, and ***diversity*** in ecology. It reflects the number and balance of elements in a closed system. Kumar (1987) sees the immediate parallel to flexibility as the ***number*** and ***freedom of choices***. From a decision theoretic perspective, Pye (1978) associates entropy with the uncertainty in a decision maker's future moves as retained in the amount of flexibility. In energy policy, Stirling (1994) uses entropy as an index for diversity, which is an aggregate or portfolio view of flexibility. Long established and widely used, entropy seems to offer a way of measuring flexibility.

This section investigates the properties of entropy (section 7.4.1) that make it attractive as a measure of flexibility, as suggested by Pye (1978) and Kumar (1987). Pye treats flexibility as the uncertainty of future decision sequences, and transitions are ***irreversible***. Kumar applies flexibility to manufacturing systems, where state transitions are ***reversible***. Pye uses logarithm to the base 2 while Kumar uses the natural logarithm. Their arguments are theoretically based though not implemented in practice. What they overlook are those properties (section 7.4.4) that make entropic measures unreliable and misleading as an indicator of flexibility. These criticisms support and expand those earlier attacks by White (1975) and Evans (1982).

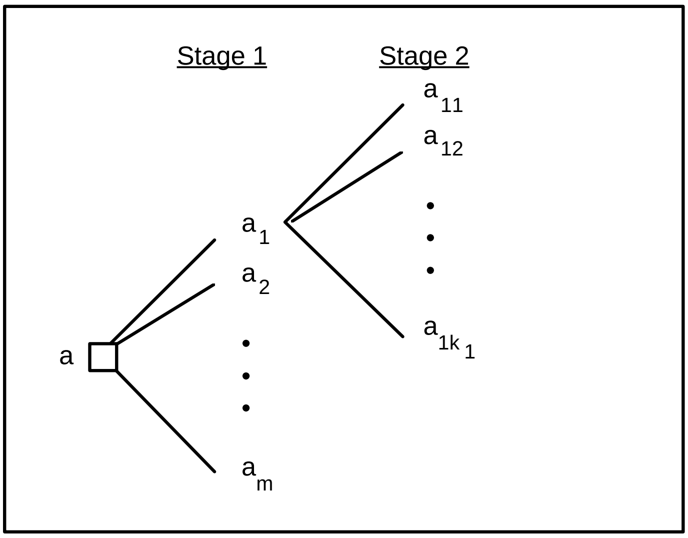
7.4.1 For Entropy

Uncertainty and flexibility are uni-directionally linked, i.e. as uncertainty increases more flexibility is required but not vice versa. A measure of uncertainty indicates the required but not actual amount of flexibility. In this context, entropy has been suggested by Pye (1978) and Kumar (1987) as a measure of the number of choices and the freedom of choice. Freedom of choice can be interpreted as the probability of selecting a particular option.

The basic formula for entropy is the negative sum of logarithms of the probabilities weighted by the probabilities: $H(\boldsymbol{a}) = -\sum_{i=1}^{m} p(\boldsymbol{a}_i)LN(p(\boldsymbol{a}_i)) = \sum_{i=1}^{m} p(\boldsymbol{a}_i)LN(1/p(\boldsymbol{a}_i))$

where $\boldsymbol{a}_i$ are states of the uncertain event following the previous node $\boldsymbol{a}$, which is an initial position that leads to a situation of *m* states as depicted in figure 7.10. In the one stage case H($\boldsymbol{a}$) = S($\boldsymbol{a}_1$, $\boldsymbol{a}_2$, ... , $\boldsymbol{a}_m$) and the above formula applies. Two or more stages can be computed using the decomposition rule explained in 5) below. The number of states n($\boldsymbol{a}$) = m. The probability that the $\boldsymbol{a}_i$ th state occurs is represented by p($\boldsymbol{a}_i$). The basic formula has the property that 0 ≤ H($\boldsymbol{a}$) ≤ LN(n($\boldsymbol{a}$)).

Figure 7.10 Notation

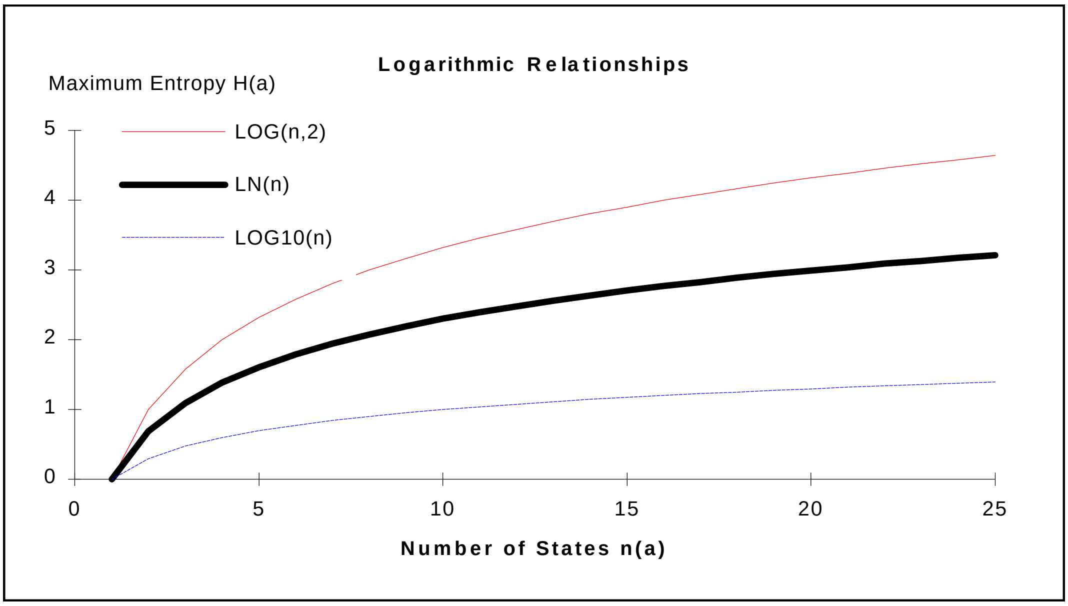


Below are properties of entropy that reflect our notion of flexibility.

1) Entropy is zero if all states except one have zero-probability, otherwise it is positive. If we only have one choice, then we have no flexibility. LN(1) = 0.

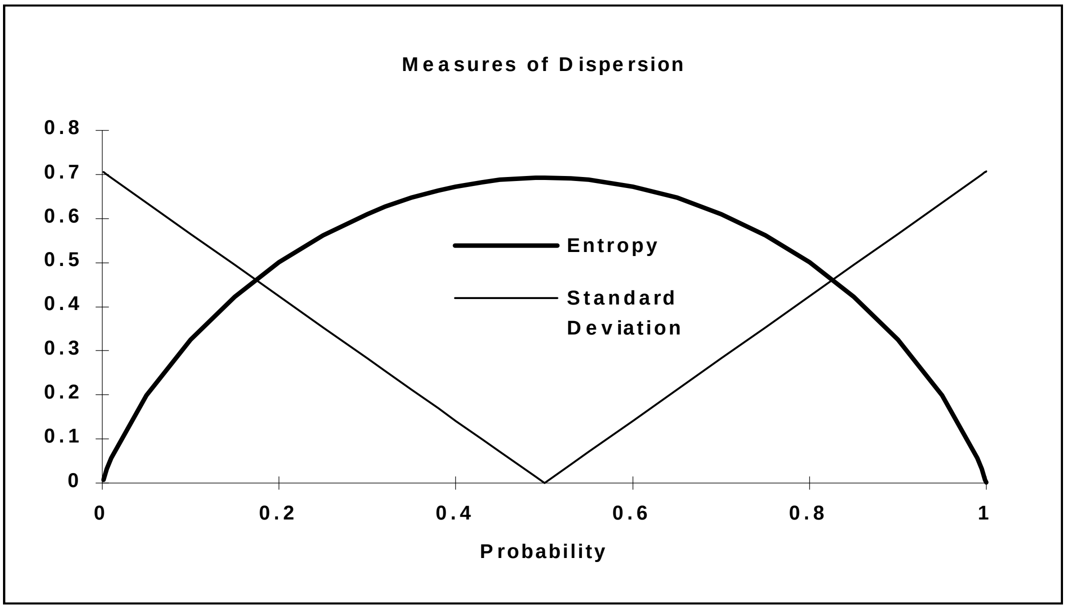
2) Entropy increases with the number of states. The more choices we have, the more flexibility. Figure 7.11 shows the logarithmic relationship between the number of states n($\boldsymbol{a}$) and entropy H($\boldsymbol{a}$).

Figure 7.11 Maximum Entropy as a Function of States



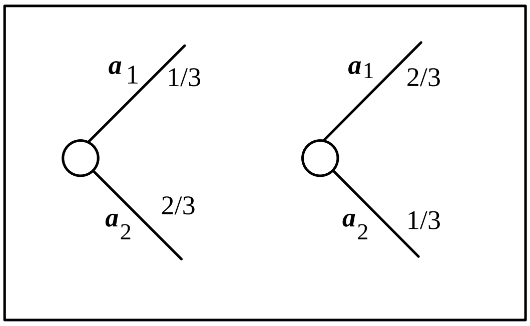
3) Maximum entropy occurs if each state has equal probability. For any ***a*** with m states, p(***a***$_i$) = 1/m for all i = 1 to m. H(***a***) = LN(n(***a***)). We have maximum flexibility if each choice has an equal chance of being selected. For n = 2, figure 7.12 shows that entropy is greatest when p = 0.5 where standard deviation of the distribution is 0.

Figure 7.12 Entropy and Standard Deviation



4) Entropy is a symmetric function. If the state probabilities are permuted amongst themselves, the entropy will not change. That is, S(1/3, 2/3) = S(2/3, 1/3) as in figure 7.13. Similarly S(1/6/,1/6,1/2,1/6) = S(1/2,1/6,1/6,1/6). This means that entropy is independent of value or payoff and does not distinguish between states. Entropy depends only on the total number of states and the permutation of probabilities.

Figure 7.13 State Discrimination

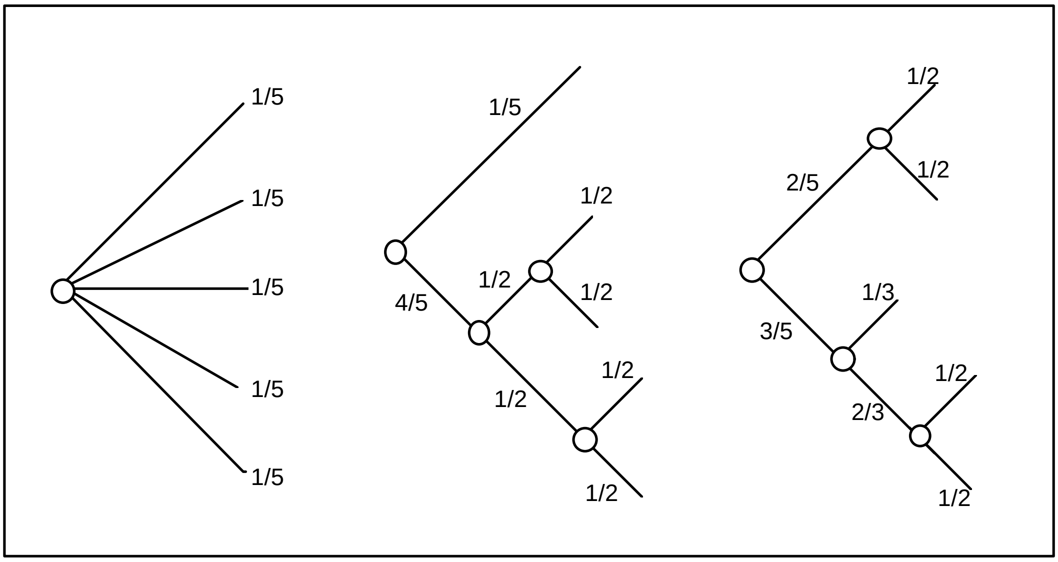


5) The decomposition rule states that every multi-staged tree can be reduced to its equivalent one stage multi-state form. The three probability trees in figure 7.14 have equal entropies of LN(5). They follow the decomposition formula of between groups and within groups. Within a group, the basic formula applies. Between groups they are additive.

$$H(\boldsymbol{a}) = S(\boldsymbol{a_1}, \boldsymbol{a_2}, \ldots , \boldsymbol{a_m}) + \sum_{i=1}^{m} p(\boldsymbol{a_i})H(\boldsymbol{a_i}) \text{ where } S(\boldsymbol{a_1}, \boldsymbol{a_2}, \ldots , \boldsymbol{a_m}) = \sum_{i=1}^{m} p(\boldsymbol{a_i})LN(1/p(\boldsymbol{a_i}))$$

$H(\boldsymbol{a_i}) = 0$ if $\boldsymbol{a_i}$ are terminal nodes. The first tree is single staged, so the basic formula applies: $H(\boldsymbol{a})$ = 5*1/5*LN(5) = LN(5). The entropy for the second tree is 1/5LN(5) + 4/5LN(5/4) + 4/5[LN(2) + 1/2LN(2) + 1/2LN(2)]= 1/5LN(5) + 4/5LN(5/4) + 8/5LN(2) = LN(5). The entropy for the third tree is 2/5LN(5/2) + 2/5LN(2) + 3/5LN(5/3) + 3/5[1/3LN(3) + 2/3LN(3/2) + 2/3LN(2)] = LN(5).

Figure 7.14 Decomposition Rule



6) Entropy increases if the states are brought closer together, i.e. probabilities becoming more equal. The more indifferent we are to the choices or the more equally favourable the choices are to us, the greater the flexibility. In this sense, entropy is also a measure of dispersion. For example, S(1/7, 6/7) < S(2/7, 5/7) < S(3/7, 4/7) < S(1/2,1/2). Any averaging procedure that brings the probabilities closer together will increase entropy.

7) Entropy is continuous and differentiable, as evident from figure 7.12. A maximum can be found. It is therefore more attractive than the size of choice set measure, which is discrete, and other measures of dispersion, like standard deviation, which are not uniformly continuous or differentiable.

7.4.2 Decision View (Pye, 1978)

Pye's notion of flexibility reflects our basic understanding: ***the ability to adapt or be adapted to changing circumstances***. His treatment of flexibility as ***the amount of uncertainty which the decision maker retains concerning the future choices he will make***, however, deviates from the expected value treatment of flexibility. By considering all future decisions as uncertain, he uses entropy to measure this uncertainty. Since the basic entropy formula contains probabilities ***not values***, he suggests two ways of incorporating value: ***weighting value as probabilities*** or ***using a cut-off satisfactory value level to reduce the total number of choices***.

He further defines robustness as a method of trading off flexibility against expected value and proposes entropic measures for three types of robustness.

1) Pye's first robustness measure reflects the independence of flexibility and value. "The most robust move is that which retains maximum flexibility." Maximum uncertainty occurs when all moves are equally likely, in which case, entropy is simply the logarithm of the number of moves. For any multi-staged probabilistic tree, the decomposition rule applies.

2) The second robustness measure is based on Simon's (1964) satisficing model. "The most robust move is one which retains the most flexibility subject to the condition that the decision maker's estimate of the probability of choosing a future sequence of moves depends on its value." The value associated with each move is translated into a probability of the move by weighting the value by the sum of all values. ***It is intended for multi-dimensional values that cannot be linearly combined into a single utility function. However, Pye does not show how to combine these values otherwise.***

3) The third robustness measure combines probabilities and values using a cut-off satisfactory level. If the value is above the cut-off level, the difference is taken and weighted accordingly. Any value below this level is not included in the final weighting.

There are five problems with the practical implementation of this theoretical treatment.

1) The first robustness measure works only with equal probabilities. If the number of choices are reduced, the remaining probabilities must be adjusted to equal probabilities rather than re-weighted. In practice, we would re-weight the remaining choices and get unequal probabilities.

2) The second robustness measure weights the individual values by total values and performs expected value on probabilities and values. These two rounds of averaging distorts the real picture as entropy will undoubtedly increase.

3) The usefulness of the second measure for values that cannot be linearly combined eludes the reader as Pye does not discuss the method of value combination either before or after the weighted probability transformation.

4) Negative values cannot be weighted into probabilities, but Pye does not indicate whether they should be rescaled to positive values or dropped from the calculation.

5) The third robustness measure combines aspects of the former two measures but ignores the imbalance of re-weighted and rescaled value-transformed probabilities.

In addition to the above criticisms, Pye touches upon four issues that are controversial to our understanding of flexibility. The first paradox concerns information, value, and flexibility. The second paradox surrounds decision analysis and entropic treatment of flexibility. The third issue is about value and flexibility trade-off. The fourth issue concerns the transformation of values into probabilities. These issues are discussed below.

1) Pye states: "the introduction of information about the value of sequences of moves reduces the decision maker's generally desirable uncertainty concerning his future moves and so reduces flexibility." This means that the decision maker retains maximum flexibility when no values are known and all future moves are equally likely. Thus the size of choice set is maximally large and maximally uncertain. As soon as the value of any choice is revealed, the set becomes differentiated and flexibility is reduced. However, Merkhofer and others state just the opposite. Any new information resolves some degree of uncertainty before the decision is made and consequently aids the decision maker in improving upon the outcome. The value of this information depends on the degree to which the final payoffs can be effected. Any information about value should help the decision maker assess the amount of flexibility he has. These two views of information, value, and flexibility reflect a paradox: ***value of information versus information about values*** and the ***effect on flexibility*** and the ***value of flexibility***. Pye's statement suggests that flexibility is inversely related to value, but this contradicts the size of choice set definition of flexibility, i.e. the number of favourable (valuable) choices.

2) A second paradox ***concerns ideological differences between decision analysis and flexibility.*** Pye recalls: "in classical decision analysis, value is maximised and a dominated move would be eliminated from the set of moves under consideration on the basis of estimates of value made at the time of the initial decision. When maximising flexibility it is most inappropriate to eliminate a move, since the fewer are the moves, the smaller is flexibility, unless the sequence of moves is rejected as unsatisfactory." Decision analysis uses dominance of values to eliminate unfavourable moves, while Pye's entropic treatment of flexibility leaves options open. This paradox demarcates two ways of thinking: flexibility and favourability. However, flexibility has no value without considering the favourability of options, which is necessary to differentiate between the choices and eliminate the less favourable ones.

3) A third contention concerns Pye's definition of robustness, that it trades off flexibility and value. In our conceptual framework, we define robustness as tolerance against uncertainty, implying no need to change. His method of trading off the number and equal probability of choices (as contained in maximum entropy and maximum flexibility) against value by means of a satisfactory cut-off level, reduction of choice set, and re-weighting ***oversimplifies*** and ***overlooks*** the real issues in trading off flexibility and favourability as studied by Mandelbaum and Buzacott (1990).

4) Finally, Pye's method of linearly weighting values as probabilities assumes that ***the probability of a move is linearly dependent on the value of the move***. He does not address the implications of eliminating unfavourable moves and rescaling values. Any averaging method tends to increase entropy, and this is misleading for value optimisers. Furthermore, the transformation of values to probabilities assumes that the relative importance of values is the same as the freedom of choice. The relative importance of a choice is not necessarily the same as the likelihood of selecting it. Other factors such as the disablers of time, cost, and effort should be taken into account in determining the freedom of choice. Even so, such value-probability weighting does not transform the underlying process into a stochastic one, which is the essence of entropy.

7.4.3 Systems View (Kumar, 1987)

Originally, the entropy concept was applied to a closed system, from which two views emerge and often coincide. One concerns the uncertainty of selecting an element from a system. The other concerns the transformation of the system into a different state. Pye and Kumar do not distinguish these two system views nor the shift into decision terminology whence states become choices and options. In applying entropy to manufacturing systems, Kumar equates the states of a system with choices of workstations. Kumar (1987, p. 958) defines "the flexibility in action of an individual or a system depends on the decision options or choices available and on the freedom with which various choices can be made." Entropy is exactly such a function of the ***number of choices*** and the ***freedom of choice***.

Kumar (1986) applies entropic concepts from a Markovian system (with reversible state transitions) to measure the loading flexibility and operations flexibility of a Flexible Manufacturing System. The decomposition property of entropy, which enables the addition of ***between group*** and ***within group*** entropies, is attractive for measuring the operations entropy of workgroups. Later, Kumar (1987) develops (but does not apply) four entropic measures to satisfy desirable properties of flexibility measures.

Kumar claims that entropy offers an objective basis for measuring flexibility but does not mention the way these probabilities are derived. The probability associated with state $\boldsymbol{a}_i$ (reflecting the probability that $\boldsymbol{a}_i$ is chosen) is the same as the transitional probability from $\boldsymbol{a}$ to $\boldsymbol{a}_i$. These transitional probabilities are not the same as the probabilities associated with the states of the trigger event. Kumar's probabilities reflect the freedom of choice, which seem to encompass our indicators of disablers and motivators as reflected in the proportion of favourable outcomes, availability of options, probability distribution of the trigger event, and other factors that affect the likelihood of choosing the second stage option.

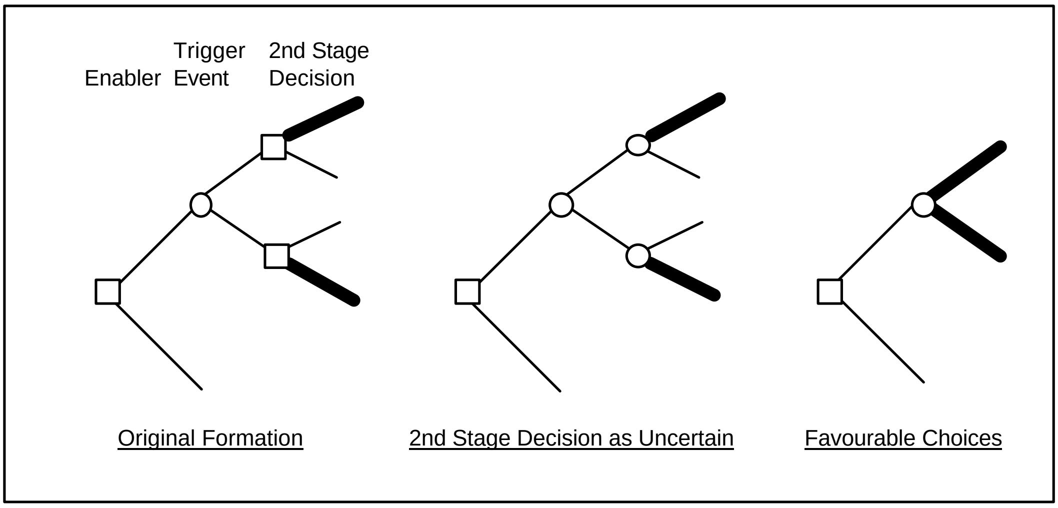
Although entropy's attractive properties reflect our notion of flexibility, the treatment of values, the derivation of probabilities for freedom of choice, and the rules for determining which choices to include have not been addressed by Kumar. To investigate this further, we consider different ways of deriving these probabilities: transforming values such as switching cost and response time into probabilities or using subjective probabilities. The conclusion is the same: no matter how sophisticated the method of probability derivation, entropic measures cannot distinguish between states. Therefore, ***any entropic measure will not be able to trade-off value and probability***. While it may be true that the greatest flexibility exists when all choices are equally likely, entropy fails when the choices are not equally likely. That is, it is possible to have $H(\boldsymbol{a}) > H(\boldsymbol{b})$ when $n(\boldsymbol{a}) < n(\boldsymbol{b})$ because choices are not equally likely. These fundamental problems with entropy are listed in the next section.

7.4.4 Against Entropy

Entropy was introduced to operations research and systems modelling in the sixties. A series of pro-entropy and against-entropy articles appeared in the OR literature: Dreschler (1968), White (1969), Wilson (1970), White (1970), and White (1975). These, along with an excellent review by Horowitz and Horowitz (1976), seemed to have sealed the fate of entropy in this field. As these criticisms were not specific to flexibility, entropy received another comeback in Pye (1978) and Kumar (1987). While heralding the properties of entropy, Pye and Kumar have glossed over fundamental issues, such as the derivation of probabilities and the importance of state discrimination. This section discusses these issues and their implications for flexibility measurement.

1) Entropy concerns uncertainty, at best, probabilities. Flexibility is about choices, which are states of a decision. The selection of choices depends on decision rules of value optimisation and trade-offs of decision criteria. To use entropic measures, we need to represent these decisions either as uncertainties or associate them with the trigger event, as in figure 7.15. If they are represented as uncertainties, the decision rules that determine their selection must somehow be transformed into probabilities. However, if all future decisions are treated as uncertain, there is no way of distinguishing between choosing or not choosing other than probabilities of 1 and 0, in which case it reduces to a trivial configuration.

Figure 7.15 Decision Tree Transformation for Entropic Treatment



2) Entropy does not give any more information than the permutation of probabilities (reflecting balance or dispersion) and the total number of states. If the same probabilities are reassigned to different states, the entropy remains the same. Adding new states with zero-probabilities does not change the entropy. Thus any permutation or addition of zero probability states will not change the entropy.

3) Entropy does not discriminate between states. Any re-assignment of the same discrete probabilities to different states will give the same entropy. For n = 2, it is not possible to distinguish between choosing or not choosing.

4) As the number of non-zero probability states increases, entropy will increase. Intuitively, as the number of stages increases, entropy should also increase. The attractive decomposition property of entropy, however, implies that multiple stages may not necessarily give higher entropy than single or fewer stages.

5) H(***a***) is always greater than H(***b***) if n(***a***) > n(***b***) and p($\boldsymbol{a}_i$) = 1/n(***a***) and p($\boldsymbol{b}_i$) = 1/n(***b***). Without equal probability, H(***a***) is not necessarily greater than H(***b***). As such, entropy is not a reliable indicator of number of choices.

6) It is meaningless to use entropy in the expected value manner, i.e. to weight and aggregate them because the maximum entropy of equal probabilities is no longer apparent. That is, E(H(***a***)) = - $\sum_{i=1}^{m}$ p($\boldsymbol{a}_i$)v($\boldsymbol{a}_i$)LN(p($\boldsymbol{a}_i$)), where v($\boldsymbol{a}_i$) is the value or payoff associated with the $\boldsymbol{a}_i$th state.

7) Entropic calculations do not reflect any dependence of probabilities. Entropy remains the same regardless of dependence relationships as long as all multi-stage trees decompose to the same permutation of probabilities in the same number of states. Several dependence relationships are thus ignored: state, probability, value.

8) Even as a measure of dispersion, it does not provide more information than standard deviation, which takes value assignments into consideration. For example, S(1,1,1,2,3) = S(2,2,2,4,6) = S(0.125, 0.125, 0.125, 0.25, 0.375) = 1.494. Standard deviation of (1,1,1,2,3) = 0.894. Standard deviation of (2,2,2,4,6) = 1.789 which is twice the previous standard deviation.

9) Cumulative probability is meaningless in the entropy context. Any regrouping within a stage by cumulative probabilities will change the entropy because the number of states and permutation of probabilities change. For example, S(1/3,1/3,1/3)>S(2/3,1/3) even though two of the three states may be similar.

10) Entropy is an absolute measure, not relative. Flexibility is relative with respect to events and decisions, states and choices, probabilities, and values. These additional value indicators are simply not considered in entropic calculations.

11) Transforming values into probabilities for entropic calculation is not as straightforward as Pye has described. As negative values are not allowed, they must be dropped or rescaled. If they are dropped, entropy is reduced. Additive rescaling (adding all values by a constant) changes the probabilities and entropy but does not change the standard deviation. Any averaging procedure brings probabilities closer together hence distorting the original dispersion and balance. Transforming values into probabilities does not make it a stochastic process. These state probabilities depend on how easy, how favourable, and how likely we are to choose it compared to other options available from the same decision stage. Such value-probability transformation confuses likelihood with relative importance.

12) The concept of entropy was originally developed for systems not decisions. The system (state transition) and decision (stages) views are quite different. State transitions in systems are reversible while most decisions are irreversible and the choices are mutually exclusive, in-so-far as decision analysis is concerned.

13) The decomposition property separates the entropy between groups and the entropy within groups. It is meaningful for the systems view but not the decision view. For the decision view, stages represent passage of time, whereas in the systems view, stages are only state transitions not chronological sequences. In decision sequences, the number of stages reflect not only passage of time (the more stages, the more time) but also conditionality and additional costs.

14) Any re-order or reversal of independent chance events in a symmetric sequence produces the same entropy. That is, H(A, B, C) = H(C, A, B) = H(A, C, B) = H(B, A, C) where A, B, and C are independent chance nodes. There is no concept

of value of information as pertained to the early resolution of uncertainty, because symmetric reversals give the same entropy. In this respect, White (1975) observes correctly that entropy fails to relate information to its uses.

15) Entropy is not a unique measure. Table 7.4 shows different distributions of five, four, and three states each giving the same entropy 1.099, while probability assignments as well as standard deviations differ in each case. A linear transformation of values into probabilities will not give a unique entropy.

Table 7.4 Equal Entropies for Different Number of States

States	v(a_i)	P(a_i)	H(a_i)	v(a_i)	P(a_i)	H(a_i)	v(a_i)	P(a_i)	H(a_i)
a_1	1	0.0388	0.1262	1	0.0456	0.1409	5	0.3333	0.3662
a_2	1	0.0388	0.1262	3	0.1369	0.2722	5	0.3333	0.3662
a_3	1	0.0388	0.1262	6	0.2738	0.3547	5	0.3333	0.3662
a_4	10.7476	0.4174	0.3647	11.9150	0.5437	0.3313			
a_5	12	0.4661	0.3558						
entropy			**1.099**			**1.099**			**1.099**
standard deviation	5.6992	0.2214		4.7936	0.2187		2.7386	0.1826	

Some of these criticisms have broader foundations, as noted by Horowitz and Horowitz (1976) below in 16 and 17.

16) Entropy is based on the analysis of a stochastic process. Decisions are not stochastic. Weighting values so that they sum to one does not make the process stochastic. There is no observational or theoretical basis for the analogy between physical and economic processes.

17) Entropy can be derived from parameters of moment distributions, so it does not contribute any additional information.

The above arguments suggest that entropy is not suitable for our conceptual treatment of flexibility where decisions reflect value optimisation. Evans (1982) points out that Pye's model can lead to bad decisions. Entropic measures are only applicable to situations where preferences can be quantified and changed into probabilities. Entropy is overwhelmingly dictated by the size of choice set and non-trivial dispersion of the probability distribution while completely excluding payoffs. Although entropy promises a precise, coherent, and objective measure, it does not capture the multi-dimensional aspects of flexibility. It may be useful as an overall indicator of number of choices and balance (dispersion) in situations where there are many choices and many stages, thus necessary to prune the tree before further analysis. However, entropy cannot deal with the trade-off between balance (freedom of choice) and number of choices unless these choices are equally likely or equally attractive, in which case it is just as easy to count them. If maximum flexibility is reflected by equal probabilities, then the decision maker must be indifferent to the choices, meaning that all choices are identical. This goes against Mandelbaum's (1978) diversity as a source of flexibilty. Diversity increases flexibility because it increases the capability of responding to different conditions. But entropy decreases as diversity (of probability assignments not value) increases.

7.5 Comparison of Entropic and Expected Value Measures

Entropy and expected value are methods of weighting and aggregating probabilities. Both have been proposed as measures of flexibility.

The recursive equations of entropy and expected values for a multi-staged tree can be set such that H(***a***) and E(***a***) are equal.

$H(\boldsymbol{a}) = S(\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots, \boldsymbol{a}_m) + \sum_{i=1}^{m} p(\boldsymbol{a}_i)H(\boldsymbol{a}_i)$ where $S(\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots, \boldsymbol{a}_m) =$

$\sum_{i=1}^{m} p(\boldsymbol{a}_i)LN(1/p(\boldsymbol{a}_i))$

$H(\boldsymbol{a}_i) = 0$ if $\boldsymbol{a}_i$ are terminal nodes.

$$E(\boldsymbol{a}) = \sum_{i=1}^{m} p(\boldsymbol{a}_i)E(\boldsymbol{a}_i) \text{ where } E(\boldsymbol{a}_i) = \sum_{j=1}^{k_i} p(\boldsymbol{a}_{ij})E(\boldsymbol{a}_{ij}).$$

$E(\boldsymbol{a}_{ij}) = V(\boldsymbol{a}_{ij})$ if $\boldsymbol{a}_{ij}$ is a terminal node. $V(\boldsymbol{a}_{ij}) = v(\boldsymbol{a}_i) + v(\boldsymbol{a}_{ij})$ [all values along the path to the final node.]

$H(\boldsymbol{a}) = E(\boldsymbol{a})$ if $v(\boldsymbol{a}_i) = LN\ (1/p(\boldsymbol{a}_i))$ or $-LN(p(\boldsymbol{a}_i))$

Entropy and expected values are equal for a probabilistic decision tree (no decision nodes) if values and probabilities are related in a logarithmic manner.

Other than the above commonalities, expected values and entropy have entirely different theoretical assumptions. Expected value is a statistical measure whereas entropy is not. Entropy is simply a continuous and differentiable measure of the number of choices and dispersion of probabilities. It appeals to our notions of flexibility because of its decomposition and other properties. However, the arguments against entropy in section 7.4.4 suggest that simpler measures of dispersion and proportion (cumulative probability) of favourable choices and number of favourable choices may be more apparent and more widely applicable.

Measures of flexibility depend on definitions of flexibility. Each author cited in this chapter defines flexibility differently. As such, they make use of different indicators of flexibility. Pye defines flexibility as the uncertainty regarding decision maker's moves. Hobbs defines flexibility as choices available to respond to uncertainty, where a first stage decision guarantees the certainty (availability) of future choices. Thus future decisions have no uncertainties other than the trigger events, and they are only subject to the favourability criterion. Hobbs uses enabler and motivator, but no disabler. So flexibility is positively good, ignoring the downside of flexibility as mentioned in the previous chapter. Kumar uses disabler, i.e. availability or freedom of choices and no enabler or motivator. Pye uses both disabler and motivator, but no enabler. Hobbs assumes that the first decision guarantees future flexibility, while Pye views all future decisions as uncertain, regardless of initial positions. The comparison between expected value and entropy highlights the important difference between favourability and flexibility.

Expected values and entropy appear complementary in many respects. Expected values discriminate between states because payoffs and their relative weights (as weighed by the series of probabilities associated with previous states) distinguish between states. But expected values do not indicate the number of states. Meanwhile, entropy reflects the number of states and their dispersion but does not discriminate between states. Expected values are based on the principle of dominance and elimination of sub-optimal choices. Entropy is based on the number and balance of states. Expected values mainly depend on values, with probabilities as a secondary element. Thus expected values reflect favourability. Meanwhile, entropy is value-free and does not indicate favourability.

Entropy could be a good indicator for that aspect of flexibility that is value-free and a screening device for large multi-staged probabilistic trees which treat future decisions as uncertain. However, flexibility is not value free, and the first stage guarantees flexibility in the second stage decision. Hence it is necessary to retain decisions as decisions and not uncertainties. Except for a few of its attractive properties, the information provided by entropy can be replaced by loose indicators such as the size of choice set and standard deviation. In addition to the above, various arguments in section 7.4.4 have shown the inadequacies of using entropy for our purposes.

7.6 Conclusions

Individually, each measure reviewed in this chapter does not capture the multi-faceted meaning of flexibility. We propose the combined use of indicators and expected values to measure flexibility more fully. ***Indicators*** are individually not sufficient, but together meet the criteria for measuring flexibility. ***Expected values*** appear in different forms, most of which over-emphasize the favourability aspect. We caution against total reliance on expected values and propose the use of indicators to supplement the deficiency, such as the trade-off between different aspects of flexibility. ***Entropic measures***, though possessing attractive properties, fail to meet our criteria for measuring flexibility. We dismiss any measures based on entropy. These conclusions are supported in detail below.

INDICATORS

Indicators originate from those definitional elements of flexibility. These indicators (number of choices, enabler, disabler, motivator, trigger event, trigger state, likelihood, and two stage decision sequence) describe the essence of flexibility and provide a framework for structuring options and strategies for flexibility. Not all indicators are necessary, but each indicator alone is not sufficient to capture the multi-faceted meaning of flexibility.

EXPECTED VALUE BASED MEASURES

Expected values disguise the number of options available and the uncertainty to flexibility mapping. As an aggregate measure, they emphasize the favourability aspect of flexibility. Although useful for summarising decision trees, expected values do not resolve conflicts between favourability and flexibility. Individual probabilities are aggregated but lost in the final expected value measure.

1) Relative Flexibility Benefit

Hobbs' relative flexibility benefit measures the difference between considering uncertainty as opposed to treating it as 100% probable. This measure is contingent on a mapping between the decision that realises the flexibility and the uncertain condition that provides the opportunity for it. The flexibility conveyed is that of capitalising on the maximum number of uncertain events. It differentiates between flexibility and no flexibility, but not the degree of flexibility. It does not resolve the conflicts between the flexibility and favourability of different investments.

2) Normalised Flexibility Measure

This measure depends on the "matchability" or calibration between trigger states and second stage options. A poor match gives a large slack and less favourable outcome than the tighter fit of a better match between the states of the condition and the options available. The initial choice that leads to minimum total slack gives the most flexibility. It provides an index between 0 and 1, and is useful for comparing different degrees of flexibility.

3) Expected Value of Information

The expected value of information explains the structuring of trigger events before decisions that give flexibility. Defined for decision flexibility, it pertains only to postponement of decisions.

4) Towards an Improved Measure: Weighted Normalised Measure

In an attempt to overcome deficiencies of existing expected value measures, we investigated the use of EVPI and Deterministic EV to ***anchor*** the normalisation and a cumulative probability distribution of the trigger event to ***weight*** the normalised measure. Although the resulting ***weighted normalised measure*** performs better than existing expected value measures in our extension of Hobbs' example, we caution its use for the following reasons.

1) EVPI and Deterministic EV requires restructuring the decision tree, which may not be possible due to conditional dependence of events. Furthermore, restructuring the tree may not be realistic, e.g. uncertainty occurs only after decision is made; no way to postpone decision or get information to resolve the uncertainty; inflexible decision structure.

2) EVPI is meaningful only in Merkhofer's terms, i.e. in the context of decision flexibility, not generalisable, thus missing out on the richness of flexibility.

3) The method of using loose indicators such as probability (likelihood) may be more revealing, simpler, and more accurate than weighting and normalisation, which tend to disguise the simple elements.

4) Further tests are required to establish its reliability and generalisability.

ENTROPY

The underlying theory behind entropy is not appropriate for our conceptual understanding of flexibility. Entropy does not discriminate between states, therefore it cannot differentiate between the favourable and the unfavourable. It does not recognise value and thus ignores the favourability inherent in flexibility. The permutations of probabilities is not unique. To use entropy, it is necessary to treat the future as uncertain, thus removing the value preferences of the decision maker.

To develop practical guidelines for the assessment and operationalisation of flexibility, we propose to

1) use indicators and expected values to measure different flexibility options and strategies, via

2) illustrative examples pertaining to the UK Electricity Supply Industry, and

3) translate Mandelbaum's sources of flexibility to this structuring framework.

We call this "***modelling flexibility***" in Chapter 8.